APPENDIX A

A SPACE-TIME ARCHITECTURAL SUPERSTRUCTURE ENABLING EFFICIENT MULTIPLE ANTENNA COMMUNICATION

By

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ABSTRACT

We consider an (M,N) wireless link, (M transmit antennas and N receive antennas), impaired by additive white Gaussian noise. The transmitter, which is subject to a power constraint does not know the outcome of the random matrix channel which has a static and flat frequency characteristic. It does know the channel statistics. The link operates at a limit on the probability of outage. We stratify diagonals in space-time to express a message for efficient communication with limited receiver complexity. The special message arrangement enables the receiver to substantially mute self interference caused by multipath, and, despite the M-dimensional transmit signal, avoid an explosion of processing complexity in the spatial domain.

We investigate examples in important downlink categories, showing that the message architecture can be extremely efficient. For all (M,1) systems it is maximally efficient. At 10% outage, and with matrix Rayleigh channels with 10dB average SNR, we see that an (8,3) and (4,2) system can operate at over 95% and over 90% of Shannon capacity respectively.

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1 INTRODUCTION

1.1 OBJECTIVE

We will be exploring (M,N) wireless communication links (M transmit antennas and N receive antennas), that are impaired by additive white Gaussian noise (AWGN). It is assumed that, when using such a channel, the transmitter is assumed not to know the spectrally flat N by M matrix transfer characteristic. The transmitter, which is subject to a power limit, transmits with equal power out of each of its M transmit antennas. We assume a "long burst" context. The view is that when the channel is used from time to time, its transfer characteristic holds constant for the communication burst, yet the channel can change significantly from one burst to the next. The transmitter only knows the channel matrix statistics. For each burst, a large number of symbols are sent, permitting the standard infinite time horizon perspective common in information theory.

The transmitter does know the channel statistics, so it can infer the distribution of channel capacities it could attain if it were privy to the

random channel outcomes. Say that the channel is allowed to be in an outage state for a small percentage, say X%, of its realizations. Without knowledge of the individual channels, the (M,N) link, can, in principle [1,2] operate at the X% capacity level for the remaining channel outcomes. In this paper the (M,N) system will be constrained by the stipulation that despite the spatially M-dimensional (M-D) transmit signal, an explosion of processing complexity in the spatial domain must be avoided by communicating with parallel one dimensional (1-D) codecs. As discussed in Section 2.0, other basic implementation concerns will be kept in mind as well.

We will suggest a means for expressing signals so that the encoded message is disposed in space-time to enable the receiver to substantially mute self interference. After describing the communication architecture in Section 2.0, we analyze it in Sections 3.0 and 4.0 and then study it numerically for the case of a matrix Rayleigh channel in Section 5.0. We will see evidence, that such an architecture, when used in certain important downlink categories, like when M is substantially in excess of N, can operate at a specified outage level with near maximum efficiency.

The study of (M,N) systems is a very active area of research: [3-11] are sample references. Additional related papers are cited as we proceed. In contrast to many of the references, we are primarily concerned with a message's architectural superstructure so that the specific coding and modulation used in the fundamental 1-D space-time building blocks that we will introduce will not be our direct concern. A few of the references do deal with architectural superstructures: [12] is concerned with general (M,1), while [13-14] and [15], investigate (2,1), and (4,1) respectively.

1.2 THE VECTOR CHANNEL

We take a baseband view of an *(M,N)* wireless communication link. The transmitted *M*-D vector signal has components denoted $s_i(t)$, $i = 1,\ldots M$ that are nominally complex, statistically independent white Gaussian signals of equal power, *P/M*. So the total radiated power is *P*. These *M* components of a vector process, $s(t)$, are transmitted over a noisy, spectrally flat, *N* by *M* complex matrix channel, *G*, that can cause the *M* transmitted signal components to interfere with each other. This self-interfering means of communication, is described by the following vector equation for the received *N*-D signal, $r(t)$ in terms of $s(t)$

$$r(t) = Gs(t) + \nu(t). \tag{1.1}$$

The *N*-D vector, $\nu(t)$, represents the complex additive white Gaussian noise (AWGN) impairment. We assume that $\nu(t)$ is both temporally and spatially white. For convenience we will often take time to be discrete, with each clock tick corresponding to the time it takes for exactly one coded vector symbol to be received (or alternately, sent).

When convenient for simplifying our analysis we can redefine the vectors in (1.1) to be normalized. Then, each vector component is divided by the standard deviation of an additive noise component, σ, so that all the noise variances become unity and the normalized signal power radiated from each transmit antenna becomes $P/(M \cdot \sigma^2)$ instead of *P/M*.

The signal $s(t)$ has spatially and temporally white Gaussian components representing the limit of a bandwidth efficient, forward error protected, encoded vector signal. We will architect the space-time

disposition of various basic building blocks that are put together to compose this white Gaussian vector process.

The channel matrix is perfectly known at the receiver: it takes vanishingly small rate to probe the channel, so that the receiver learns it with arbitrary accuracy without detracting from capacity. The Shannon capacity of the channel described by equation (1.1) is given by

$$C = \log_2 \det\left[I_N + (P/(\sigma^2 M)) G G^{\dagger} \right]. \tag{1.2}$$

In this equation, I_N is the N by N identity matrix and det means determinant. The formula [1,2,8-10,16,17], termed the LogDet capacity formula can be derived from fundamental information theory considerations, as, e.g., appear in [18]. It holds when the transmitter only knows the value C, and not the actual entries in G. Then, with a suitably encoded signal, s, the capacity, C, can be "attained". More precisely, in the limit of a sequence of progressively more powerful codes, error free transmission at rate C can be approached to within an arbitrarily small deficit.

The LogDet capacity will be our target. Our primary interest is in exploring how close we can come in important downlink situations when additional demands are placed on the communication architecture.

2 STRATIFIED LAYERING TO REDUCE INTERFERENCE

2.1 DIAGONAL LAYERING OF SPACE-TIME

In our architectural superstructure for a received message, space is discrete with M components, $m = 1, 2, \ldots M,$ corresponding to labeling of

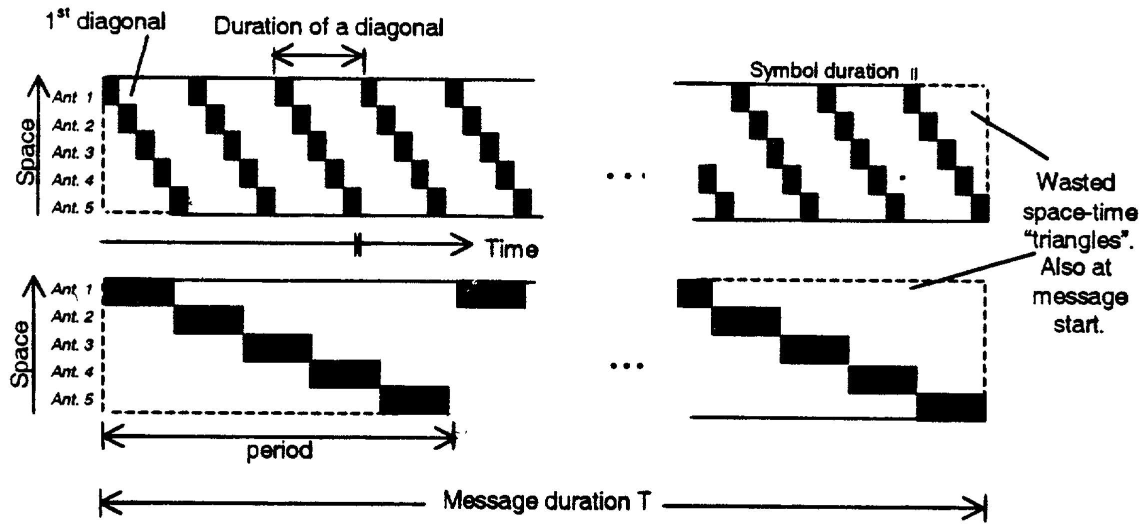
the transmit antennas. With each tick of the clock, an M-D encoded vector symbol is received (sent) consistent with equation (1.1). For the time it takes to receive (send) a coded message, we can view space-time as a rectangle. Later, in our information theory analyses, we will be interested in the limit of messages of infinite duration.

Reference [1] offered a look at a means of communication processing that used the concept of imaginary "diagonals" passing downward and to the right through space-time. Communication of this type, which effectively involves a spatially 1-D channel has been described as D-BLAST for diagonal Bell Labs layered space-time. The 1-D channel is an AWGN channel, but of the type whose noise power changes with time [19,20]. We will briefly highlight some of the ideas from [1] which we will then replace by a more refined way to utilize space-time.

Examples of diagonals are shown in Figure 1 for the case when there are five transmit antennas. These diagonals arise in describing the space-time disposition of 1-D encoded message constituents that are cycled over the antennas that they radiate from. The transmitter, not knowing the channel, does not know which transmitters enjoy stronger propagation paths over to the receive array and which suffer weaker paths, so, by cycling, the signal is hedged. These diagonals are referred to as diagonal layers (layers of space-time). Importantly, these diagonals offer a means of muting mutual interference, while enabling 1-D processing. As explained in the reference, a diagonal layer can amount to one coded block with a downward and to the right time sense. In the detection process these diagonal layers are "peeled off" one by one, left to the right. At any time, signal constituents associated with previously peeled diagonals have been removed as a source of

interference in detecting bits in subsequent diagonals. Interference from constituents in diagonals that will be peeled off later are muted using minimum mean square error processing, and, if the received signal to noise ratio is large enough, zero forcing is an option that is essentially as good.

Figure 1: Two Examples of Diagonal Layering of Space-Time for the Same Message Duration. Space is composed of five transmit antennas. Every fifth diagonal layer is shown in both figures, including the first and last diagonal layer of a message. In the lower figure each diagonal has a longer time duration and therefore wastes more space-time both at the start and end of an encoded message.



The processing highlighted above can be considered to be a version of decision feedback in space-time. As explained in reference [1], in many interesting cases C can be approached arbitrarily closely with 1-D processing in circumstances when the transmit array *does not know* the channel matrix. We mention in passing that recently this was shown to be true for all N by M matrix channels, [21]. This was done by combining D-BLAST analysis with a method used in references dealing with related cases where key information *is known* to the transmitters [22-24].

Figure 1 shows two of many possibilities for diagonalization. There are two practical reasons for preferring the short diagonal choice over the

long diagonal choice. At the start of a message, [1,2] and once again at the end, there is wasted space-time and we see the waste is less for the case of shorter diagonals. Also, in practice, the channel coherence time can be a concern if the duration of a coded block begins to approach a significant fraction of the coherence time. This concern arises because the diagonal duration approaching the coherence time undermines the assumption of a constant channel during a message. We stress that these two concerns involve practical, not theoretical, issues. They are inconsequential when one is making the idealizing assumptions that the channel is not time varying and the message is infinitely long.

For the two practical reasons just mentioned, it would appear that coding over a short diagonal is preferred. However, a countervailing practical issue arises. In practice one cannot use too short a diagonal as that does not allow adequate time to support a powerful bandwidth efficient code with forward error correcting capabilities. Also, the signal to noise ratio changes as one moves down the diagonal and that makes for a nonstandard coding environment. We will confront these issues.

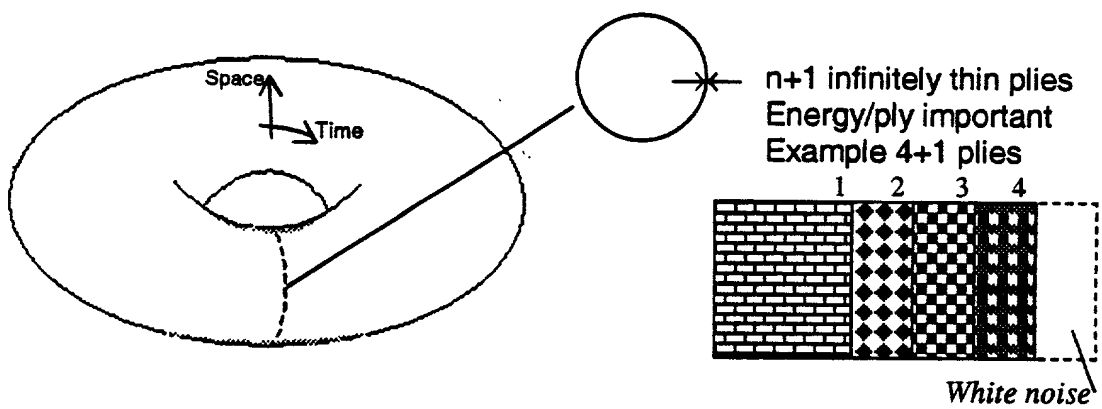
2.2 REFINED STRATIFIED DIAGONALS TO MUTE INTERFERENCE

Toward overcoming these practical difficulties, we introduce a method of communication drawing on a more refined view of space-time. We call the technique SD-BLAST for Stratified Diagonal BLAST.

As with D-BLAST, the method will involve cycling spatially 1-D coded/modulated signal constituents periodically over the M transmit antennas. In accord with this cycling we take the integer labels for transmit antennas to be the integers mod (M). It will also be convenient to view time, which is also finite, as modular. Since, in all of our analysis, we are

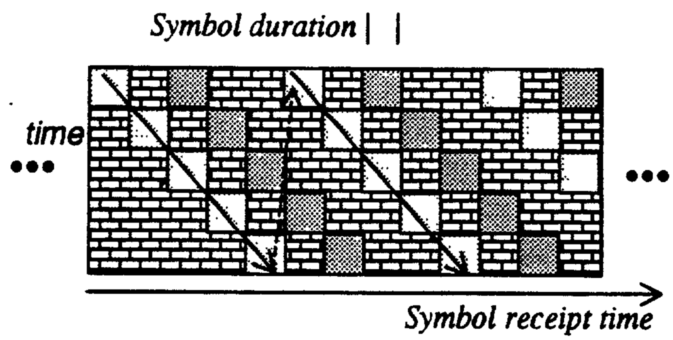
interested in the limit of long messages, it will be harmless to assume that the number of vector symbols in a message, T, is a multiple of M. So the space-time occupied by a message is viewed as the 2-D surface of a torus as shown in Figure 2. In place of successive diagonals layering space-time as in [1], we think instead of layering toroidal space-time with M helices perfectly winding around and covering the torus as noted at the bottom of the figure. In effect a long diagonal has been replaced by a helix that winds T/M times around the torus. The M congruent helices are offset from each other in time, by the time to send one symbol. These M helices also have an internal n-fold stratified spatial structure as we next explain.

FIGURE 2: VIEW OF SPACE-TIME AT RECEIVER



Each of the n plies is helically (diagonally) stratified. Loops taken by two of the five helices in the top ply are shown below for the case of five transmit antennas:

The helical (saw tooth) directed line expresses the time arrow for processing time.



It will be useful to consider the toroidal space-time surface as plied so that the surface is actually n+1 nested, infinitely thin, tori. We say that we have n+1 plies because there are n signal plies and one AWGN ply. A message will be expressed as M·n 1-D coded blocks and each block occupies exactly one out of the M helices in one of the n signal plies. The M helices which, taken collectively, perfectly wind around and pave a toroidal ply are called strata. We can think of the plies as energized by received signal constituents, except that the last innermost ply is energized by AWGN. A received signal is composed of M diagonal layers and each diagonal layer is composed of n helices, or, what is the same, n strata.

Figure 3 shows, at a high level, the way that a 5-D transmitted message corresponding to the space-time structure of Figure 2 is composed. First, a primitive bit stream is demultiplexed into five separate streams of equal rate. Then each of these streams is further demultiplexed into four substreams that are independently encoded. Later we will see that there is flexibility in choosing rates for these substreams. We will assume each of these substreams is modulated and encoded to compose four subsignals of equal power but different rates. The equal power assumption is to streamline the mathematical proof of our main result, reporting that in many cases, close to LogDet capacity can attained with 1-D codes.

For codes having a time sense, by maximizing the number of times a helix winds around the torus, all the strong path – compensates – weak path opportunities can be quickly (according to the processing clock) encountered and accomplished. This hastens the bit decision process. As already noted, diagonalization accomplishes hedging. As we shall see, the stratification feature enables significant muting of interference into any one of the *n* plies.

FIGURE 3: STRATIFIED DIAGONAL BLAST (SD-BLAST TRANSMITTER

1,6,11,16,21,26

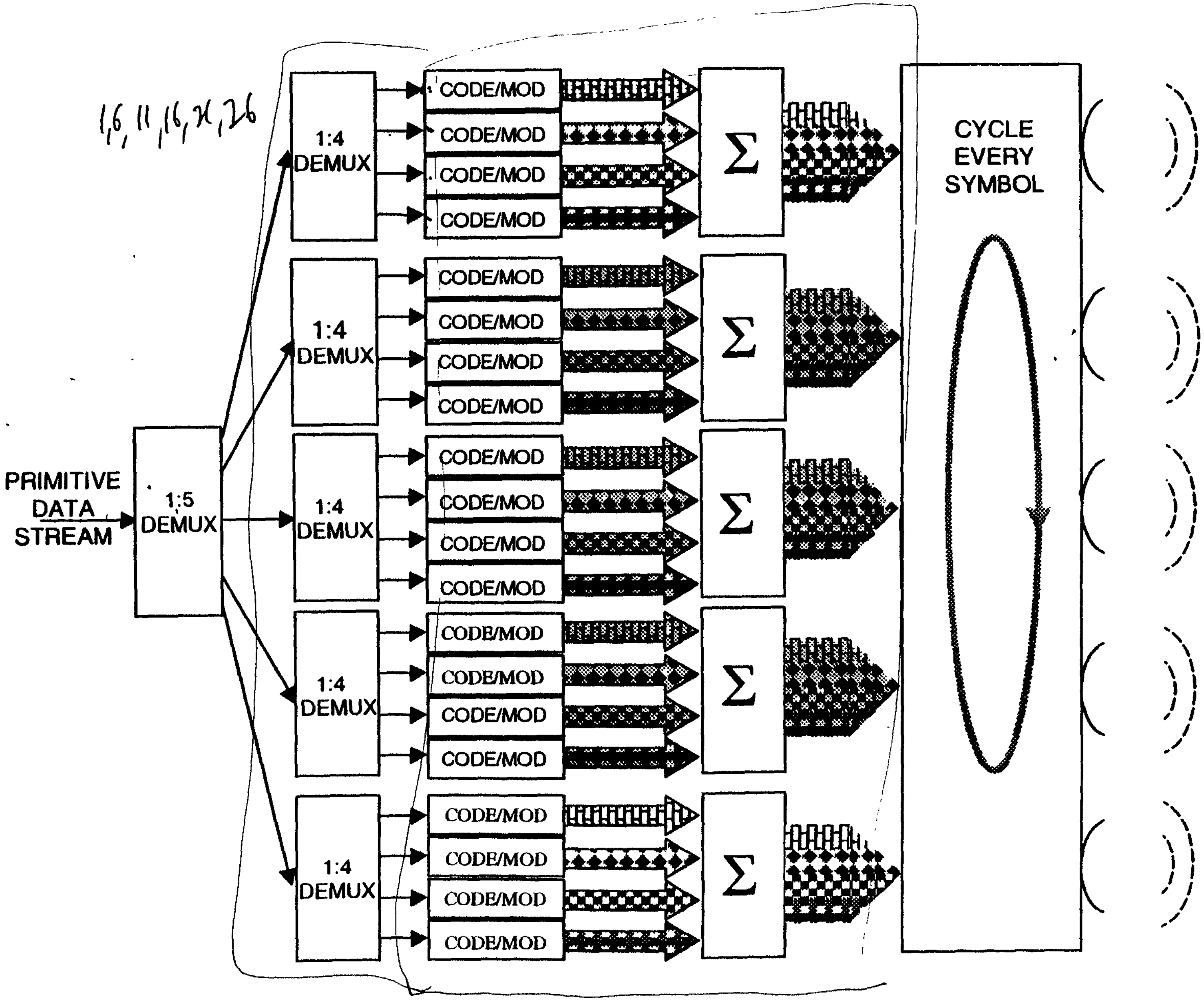


FIGURE 4: STRATIFIED DIAGONAL BLAST (SD-BLAST)

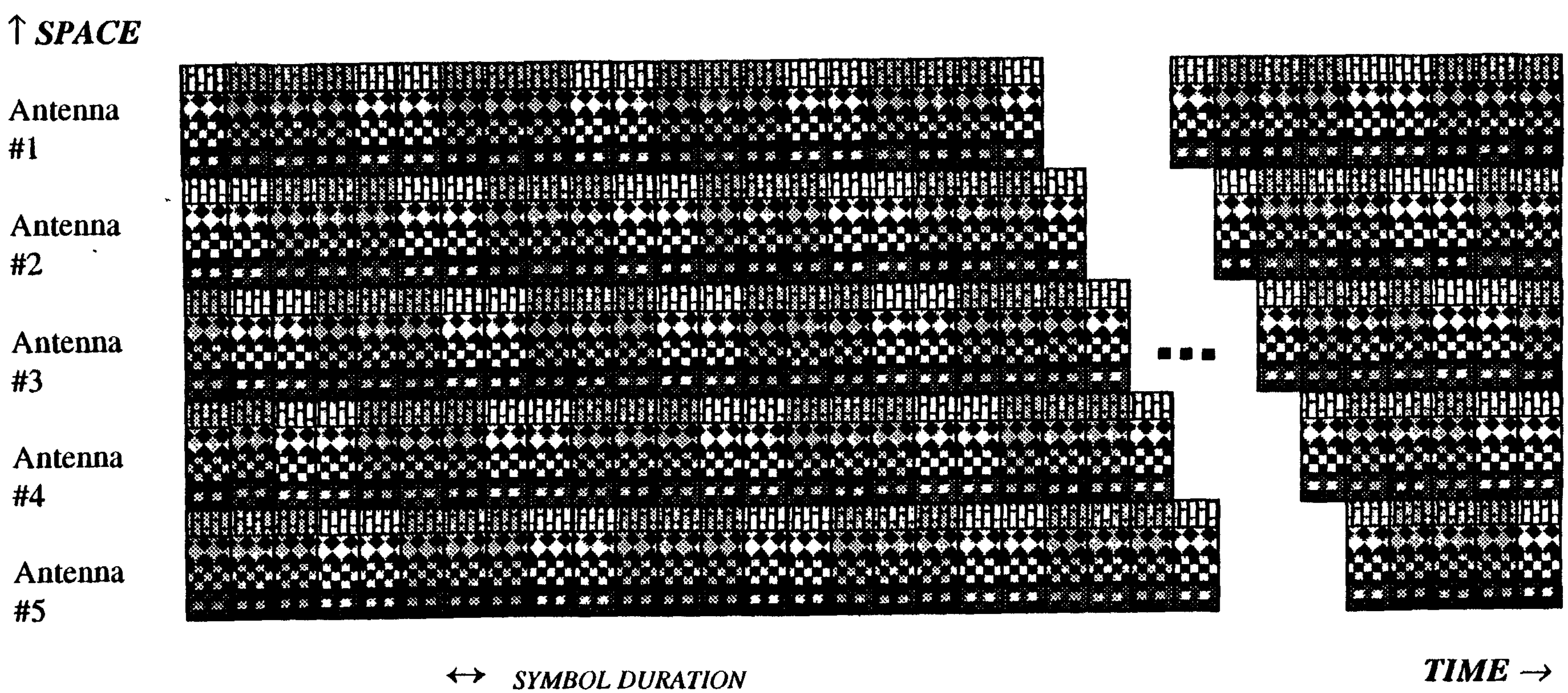


Figure 4 shows five diagonal layers, each with a different color. Furthermore, these diagonals are themselves composed of diagonal layers having the same SE directed slope. These layers within the diagonal layers are the strata. Four strata per diagonal layer are shown. Within each of the five diagonal layers, each of the four strata has a different pattern. So a total of twenty distinct strata are depicted in the Figure 4. The strata within the AWGN ply are not shown.

At this point we suggest thinking of each basic strata constituent (i.e., smallest rectangular block) coordinatized by its color and pattern at time, *t*, as merely an accommodation in space-time for a basic signal constituent. As noted in Figure 2, will also keep time with a second separate clock called the processing clock.

3 DETECTION

In this section we will give a high level somewhat qualitative description of the form of processing that is involved in deciding bits. Then, in the next section, we give an information theory analysis showing that the stratified toroidal structure can, in certain important cases "attain" or come very close to "attaining" LogDet performance.

At the receive array, the encoded bits are detected as follows. Each ply is to be thought of as an annulus like the ring of an onion slice. See Figure 5. The outer brick patterned ply, is detected first. Detection is then followed by removal of the signals in this ply as a source of interference. Prior to removal, this outermost ply, which like all the four plies hold five strata, is said to be exposed. In the process of detecting the bits in the strata in this exposed ply, each brick patterned stratum can be, but need not be,

processed simultaneously and separately. After all the bits in five strata that make up the outside ply are detected error free, the interference corresponding to this ply is then subtracted from the received signal vector. The diamond ply is then exposed. The bits in the five diamond strata are then detected and then the diamond strata signals are cancelled exposing the checkered ply, and so on, until finally the plaid patterned ply is exposed and the bits in its five strata detected.

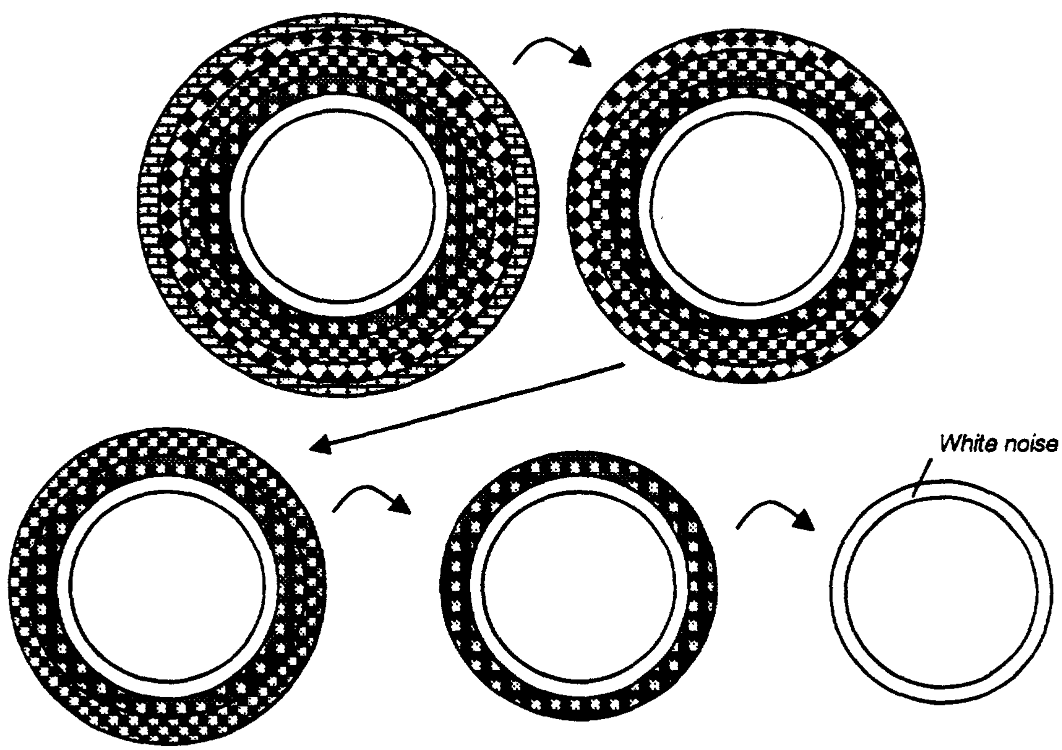


FIGURE 5: Peeling away of successive strata from outside-in in the one of colored layers. Here a layer is depicted as an "onion" with five "rings". In the sequence, peeling away another outer ring corresponds to removal of another stratum of interference. Any stratum is interfered with by all the other strata on the same ply as well by the strata that make up relatively inner plies. The peeling process can be simultaneously conducted in all five of the colored strata in a ply.

In preparing an exposed ply for detection, the N-D received vector, r(t), that was received at time t, has already been processed achieving perfect

removal of the relatively outer ply. So each inside ply is free of interference from *all* outside plies. It may be helpful to think of a sequence of processed received vectors bootstrapped with $r(t)$. So, with each major step in the processing sequence, the original received vector process $r(t)$ sheds another ply's worth of interference

$$r(t) = r^1(t) \rightarrow r^2(t) \rightarrow r^3(t) \rightarrow r^4(t) \tag{3.1}$$

where each of the four vectors is an N-D function of time defined over the message duration. Adding another level of refinement, we point out that the detection process actually includes five different copies of (3.1), one is used for each layer (color) in the example. Then a second superscript, m, is implicit for (3.1).

Each vector is then collapsed to a scalar using a weight vector, defined (up to an arbitrary complex scalar multiple), to maximize the signal to noise plus interference ratio (SINR) associated with this decision statistic. Maximization of SINR is a well known process, see, e.g., reference [2].

4 NEAR LogDet CAPACITY IN CERTAIN CASES

Next we write a mathematical formula for the bit rate that each stratum is capable of supporting if a genie informs the transmitter of the individual stratum capacities. Then we will investigate the large n asymptotics of this and related capacity formulas. The formulas that we derive will be useful in determining what can be achieved when the genie is put back in the bottle, that is, when the transmitter *is not* informed of the stratum capacities, and that is our main interest.

4.1 GENIE INFORMED STRATA CAPACITIES

We use n for the number of signal strata per diagonal layer. We derive the formula for the ultimate error free bit rate that each stratum is capable of supporting when it powered with power *P/(Mn)*. We investigate the received signal constituent, s_{mit}, on the i^{th} strata of the m^{th} component at time, t, and how it is impaired. For simplicity we proceed explicitly using only the first two coordinates. We do so for when the m^{th} strata exposed on the i^{th} ply is to be detected following the detection and removal of interference from the strata on plies *1, 2, ... i-1*. Along with AWGN, this constituent, s_{mi}, is impaired by all of the simultaneously transmitted strata with equal or higher indices that are not yet detected. The constituents of these interfering strata are $\{s_{m'l}, i \le l \le n, 1 \le m' \le M, m' \ne m \; when \; l = i\}$.

For analytical convenience we express the N by M channel matrix, G, in terms of its column vectors, so

$$G = \left(g_1, g_2, \cdots, g_M\right) \tag{4.1}$$

where g_m is an N-D vector corresponding to the m^{th} transmit antenna. Drawing on (1.1), we see the vector expression involving the received scalar signal, s_{mi}, along with its additive impairment is therefore expressed as

$$\left(g_1, g_2, \cdots, g_M\right)\left[\begin{pmatrix} 0 \\ \vdots \\ 0 \\ s_{mi} \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \sum_{l=i}^{n}{}^{(mi)} \begin{pmatrix} s_{1l} \\ s_{2l} \\ \vdots \\ \vdots \\ s_{Ml} \end{pmatrix}\right] + \nu. \tag{4.2}$$

The (mi) superscript means that the i^{th} strata of the m^{th} component is omitted in the sum, because, in (4.2), the scalar s_{mi} is signal, not interference. We are working here with the normalized form of equation (1.1), so that the N-D noise vector, ν, has complex i.i.d. Normal (0,1) components.

In equation (4.2), the impairment is the N-D vector, ξ_{mi}, is defined by

$$\xi_{mi} = G \cdot \sum_{l=i}^{n}{}^{(mi)} \begin{pmatrix} s_{1l} \\ s_{2l} \\ \vdots \\ s_{Ml} \end{pmatrix} + \nu. \tag{4.3}$$

For each value of $m = 1, 2, \cdots, M$ the variance-covariance matrix of this impairment of s_{mi} is denoted by $\Sigma_{\xi_{mi}}$. In the next step we spatially whiten this vector impairment by multiplying the expression in (4.3) by the positive definite matrix $\Sigma_{\xi_{mi}}^{-1/2}$. We get the following expression for the spatially whitened i-th stratum of the m-th component

$$\Sigma_{\xi_{mi}}^{-1/2} g_m s_{mi} + \mu^m \qquad (m = 1, 2, \cdots, M). \tag{4.4}$$

Each one of the vectors in the sequence of N-D vectors, $\{\mu^m\}_{m=1}^M$, has complex Normal(0,1) components that are statistically independent of each other. Indeed, for each value of m in a stratum independent noise is encountered. For a *(1,N)* system, where the scalar signal s_{mi} is transmitted and the N-D noisy received signal vector is described by (4.4), we know from the LogDet formula for capacity that the capacity is

$$
\begin{aligned}
c_{mi} &= \log_2\left[1+\left\|\Sigma_{\xi_{mi}}^{-1/2}\, g_m\right\|^2\cdot(P/(\sigma^2 Mn))\right] \\
&= \log_2\left[1+g_m\bullet\Sigma_{\xi_{mi}}^{-1}\, g_m\cdot(P/(\sigma^2 Mn))\right]
\end{aligned}
\tag{4.5}
$$

where the heavy dot denotes scalar product. For this *(1,N)* system, maximum ratio combining of the received vector components is a step in the processing for "achieving" Shannon capacity.

Next, employing E for expectation, we compute that

$$
\begin{aligned}
\Sigma_{\xi_{mi}} &= E\left|\sum_{j=1}^{M} g_j\left(\sum_{l=i}^{n} {}^{(mi)}s_{jl}\right)+v\right|^2 \\
&= I+\left(\sum_{j=1}^{M} g_j g_j^{\dagger}\left(\frac{n-(i+1)}{n}\right)\left(\frac{P}{\sigma^2\cdot M}\right)\right)-g_m g_m^{\dagger}\left(\frac{P}{\sigma^2\cdot M}\right)\frac{1}{n}.
\end{aligned}
\tag{4.6}
$$

We will also need the matrix identity

$$
GG^{\dagger}=\sum_{j=1}^{M} g_j g_j^{\dagger}. \tag{4.7}
$$

Later we will need the large n asymptotic form of (4.6). Consequently, for later use, we introduce z = i/n and write

$$\sum_{\xi_{mi}} = I + \left(P/(\sigma^2 M)\right)GG^\dagger(1-z) + o(1/n). \qquad (4.8)$$

Therefore, the signal to interference plus noise (SINR) of the i^{th} stratum of the m^{th} component is explicitly

$$SINR_{mi} = g_m^\dagger\left(I_N + \left(P/(\sigma^2 M)\right)GG^\dagger(1-z)\right)^{-1} g_m \frac{P}{\sigma^2 nM} + o\left(\frac{1}{n}\right) \qquad (4.9)$$

As time progresses, the ith stratum in any diagonal layer experiences an SINR that is periodic with period M. The capacity added by the ith stratum in any one diagonal layer, say, the d^{th}, $(1 \leq d \leq M)$, is then obtained by averaging over the capacity contributions from the M transmit antennas. This capacity, C_i^d, which by symmetry does not depend on d, is

$$C_i^d = \frac{1}{M}\sum_{m=1}^{M}\log_2\left[1 + g_m^\dagger \Sigma_{\xi_{mi}}^{-1} g_m \frac{P}{nM\sigma^2}\right]. \qquad (4.10)$$

This equation, which draws on formula (4.6), is a key equation that will be used for computations in Section 5.

4.2 ASYMPTOTIC FORM OF GENIE AIDED STRATA CAPACITIES

Next we derive the large n asymptotic form of the strata capacities, still with the tentative assumption is that the transmitter is made privy to these capacities. First we rewrite the previous equation as follows

$$C_i^d =$$

$$\frac{1}{M}\sum_{m=1}^{M}\log_2\left[1+g_m^{\dagger}\left(I_N+\left(\frac{P}{M\sigma^2}\right)GG^{\dagger}(1-z)\right)^{-1} g_m\frac{P}{nM\sigma^2}\right]+o\left(\frac{1}{n}\right). \tag{4.11}$$

For the large n asymptote, we employ the small ε approximation for $\log_2(1+\varepsilon)$ so that differential capacity contribution of the i^{th} stratum is expressed

$$C_i^d = \frac{P}{\sigma^2\cdot n\cdot M^2\cdot \ln 2}\sum_{m=1}^{M} g_m^{\dagger}\left(I_N+\left(P/(\sigma^2 M)\right)GG^{\dagger}(1-z)\right)^{-1} g_m. \tag{4.12}$$

In summing the incremental capacities, C_i^d, from i equal one to n, and then taking the large n limit, we get an integral over $[0,1]$. For an integral of a function of the form $f(1-z)$ over $[0,1]$, if we substitute $\zeta \triangleq 1-z$, then $\int_0^1 f(1-z)dz = \int_0^1 f(\zeta)d\zeta$. So, in the large n limit we proceed with ζ replacing $1-z$ and with $d\zeta$ associated with $1/n$. Therefore, the capacity,

$\sum_{i=1}^{n} C_i^d$, becomes the following integral

$$\lim_{n\to\infty}\sum_{i=1}^{n} C_i^d = \frac{P}{\sigma^2\cdot M^2\cdot\ln 2}\sum_{m=1}^{M}\int_0^1 g_m^\dagger\left(I_N+\left(P/(\sigma^2 M)\right)GG^\dagger\zeta\right)^{-1} g_m d\zeta. \quad (4.13)$$

Now, the detection of M such diagonal layers is executed in parallel so that the total capacity C is obtained by multiplying the right hand side of (4.13) by M. Employing *tr* for trace, after some minor linear algebra, the total capacity is seen to be

$$C=\frac{P}{\sigma^2 M\cdot\ln 2}\, tr\int_0^1 G^\dagger\left(I_N+\left(P/(\sigma^2 M)\right)GG^\dagger\zeta\right)^{-1} G d\zeta. \quad (4.14)$$

Using the singular value decomposition, [25], we write G as a triple product

$$G=U\Lambda V^\dagger. \quad (4.15)$$

Letting MIN = Min{M,N}, Λ is an M by N matrix that is all zeroes except that its NW corner is described as occupied by a MIN by MIN diagonal matrix with *jj* entry λ_j. V and U are unitary matrices sized M by M and N by N respectively. So we have

$$C = \frac{P}{\sigma^2 \cdot M \cdot \ln 2} tr\left\{ \int_0^1 \left(V\Lambda^\dagger U^\dagger\right)\left(UU^\dagger + \left(P/(\sigma^2 M)\right)U\Lambda\Lambda^\dagger U^\dagger \zeta\right)^{-1} U\Lambda V^\dagger d\zeta \right\} \quad (4.16)$$

$$= \frac{P}{\sigma^2 \cdot M \cdot \ln 2} tr\left\{ \int_0^1 V\Lambda^\dagger \begin{bmatrix} \ddots & & \\ & \left(1+\frac{P\left|\lambda_j\right|^2 \zeta}{\sigma^2 \cdot M}\right)^{-1} & \\ & & \ddots \end{bmatrix} \Lambda V^\dagger d\zeta \right\},$$

where the matrix with inverted diagonal entries that is depicted above is a diagonal matrix where for j > MIN the λ_j vanish. The trace of a square matrix is unitarily invariant, so upon carrying out the simple integration we get

$$C = \sum_{m=1}^{MIN} \log_2\left(1+\frac{P\left|\lambda_m\right|^2}{\sigma^2 \cdot M}\right). \quad (4.17)$$

Equation (4.17) can be expressed in the more elaborate form

$$\begin{aligned} C &= \log_2 \det\left[UU^\dagger + \frac{P}{\sigma^2 \cdot M} U\Lambda V^\dagger V\Lambda^\dagger U^\dagger\right] \\ &= \log_2 \det\left[I_N + \left(P/(\sigma^2 \cdot M)\right)\cdot GG^\dagger\right]. \end{aligned} \quad (4.18)$$

This is the same as the right hand side of equation (1.2). We have proved, that by knowing the strata rates, and using the communication means that we have described, in the limit of large n, LogDet capacity is "attained".

While our interest is when the strata rates are *not* told to the transmitter, we continue for now with the genie still informing the

transmitter of the ply rates. It will be useful to express the accumulation of capacity with the progressive peeling away of plies. We particularly want to do this for the asymptote of a large number of plies. We take i to be a fixed fraction, z, of n as $n \to \infty$. From equation (4.12) we see that the differential capacity added by the i^{th} ply is

$$\frac{1}{\ln 2}\cdot\sum_{j=1}^{MIN}\frac{P|\lambda_j|^2 z/(\sigma^2 M)}{1+\left(P|\lambda_j|^2/(\sigma^2 M)\right)(1-z)}dz. \tag{4.19}$$

Asymptotically for large n, the capacity accumulated through the normalized $Z^{th} = (i/n)^{th}$ ply is obtained by integrating (4.19) from 0 to Z to give

$$\sum_{j=1}^{MIN}\log_2\left[1+\frac{P|\lambda_j|^2 Z/(\sigma^2 M)}{1+\left(P|\lambda_j|^2/(\sigma^2 M)\right)(1-Z)}\right]. \tag{4.20}$$

As expected, setting Z = 1 gives equation (4.17).

4.3 PUTTING THE GENIE BACK IN THE BOTTLE WHERE IT BELONGS

As is well known, [16,26,27], a matrix channel possesses generalized eigenmodes. To access these noninterfering modes the transmitter needs to know the channel matrix. Armed with this knowledge, the transmitter can spatially water pour its available power over the modes to obtain a superior capacity than if it transmits equal power from each transmitter. Each of the summands in this equation (4.17) is referred to as an eigenrate. We are interested in the eigenrates, the rates that the (generalized) eigenmodes can support when transmitting with power P/M on each of the transmit antennas. When $M \geq N$ the channel has N eigenmodes, the interpretation of (4.17) is

that they are being blindly accessed by the transmitter, each driven with power P/M. The wasted power, (N - M)P/M, as well as the inability to water pour, is the price of channel blind operation.

We see that if all the genie does is inform the transmitter of the eigenrates then LogDet capacity can be approached in the limit of an infinite number of strata per diagonal layer. The transmitter need not know how to access the eigenmodes. When there is only one eigenmode, for example (M,1), then knowledge of C is equivalent to knowledge of the eigenrates and the genie is not necessary. (In (1,N), the single eigenmode is also known without a genie, but we are focusing on $M \geq N$.)

We will now give a general procedure, that can be carried out with a Monte Carlo method, for computing a lower bound on what this stratified space-time architecture can achieve when the genie is back in the bottle. The method is based on the statistical characterization of the channel. We stress that the transmitter is assumed privy to that and nothing more. In the next section we apply the procedure to an ensemble of random matrix Rayleigh channels and establish its effectiveness in some important cases.

We discuss how to operate at a typically small, say X%, capacity outage for an arbitrary random channel ensemble. Since the transmitter does not know the set of n bit rates per ply, it needs to make an educated guess to achieve a high value of the X%-tile outage capacity. The guessed ply rates must be low enough so that all n guesses are successful for at least (100-X)% of the channels.

Choose the guessed rates as those associated with any channel at an outage percentile Y% < X% of plies of a hypothetical channel population in which the ply rates are known. Enforce these somewhat less ambitious rates

for each channel in a population where the rates are unknown and look at the percentage of the channels that have at least one of its n plies in violation of the demanded rates. Iterate this calculation by perturbing the starting outage percentile (all lower than X%) computed from the bit rates known case until an acceptably slightly less than X% violation free count occurs with rates unknown. This assures the desired X%-tile outage capacity is met for this guessing procedure. While the outage capacity is reduced compared to the genie assisted case, in the next section we will see examples, where, it is fair to say, it can come close.

5 NUMERICAL EXAMPLES FOR MATRIX RAYLEIGH CHANNEL

Next we report some examples for the ideal matrix Rayleigh channel. We will see cases where the transmitter is not all that much in the dark as to what bit rate to use for each stratum, or equivalently, for each ply. If M >> N the M eigenrates substantially become hardened, [1,28-31], as they do when both M and N get large and M is a fixed fraction or N. In all cases, whether there is hardening or not, the iterative method of the last section is applicable.

We include results for a sampling of (M,N) systems using 10% outage, leaving comprehensive numerical studies for the future. The first two examples, (8,1) and (8,3) pertain to the important downlink case where the end user has few antennas compared to the base(s). A (4,2) and a (2,2) case are also reported. Heavy use will be made of the formulas derived in the last section. Since in all four examples $M \geq N$, we have that MIN = N.

The graphs that we present display 10% outage channel capacity versus average received signal to noise ratio (SNR). The average received

SNR, ρ, (ρ_{dB} = $10\log_{10}\rho$) is calibrated as follows. If one makes a test measurement transmitting all the available power, P, out of any of the M transmit antennas, say the m^{th}, and makes repeated statistically independent channel measurements at any receive antenna, say the jth, then ρ is given by $\rho = (P/\sigma^2)\cdot E|G_{jm}|^2$. The expectation averages over all the statistically independent instantiations of the entry G_{jm}. Note, ρ is independent of the index pair *jm*. In the SD-BLAST examples power P/M is transmitted from each of M antennas in the transmit array. All of the examples have SNRs in the range of $\rho_{dB} = -6$ dB to +24dB with 3dB steps.

Because of interest in V-BLAST, [32], which works with codes designed for a standard AWGN environment, we also include in our examples some comparative ultimately encoded V-BLAST curves. In the examples the V-BLAST system is always based on maximum SINR instead of zero forcing. It must be stressed that V-BLAST was designed for situations where N > M, so it cannot be expected to exhibit strong performance in our examples. With V-BLAST a smaller number of transmitter antennas than M can give superior performance than using all M available. Therefore, in all examples we optimize the number used.

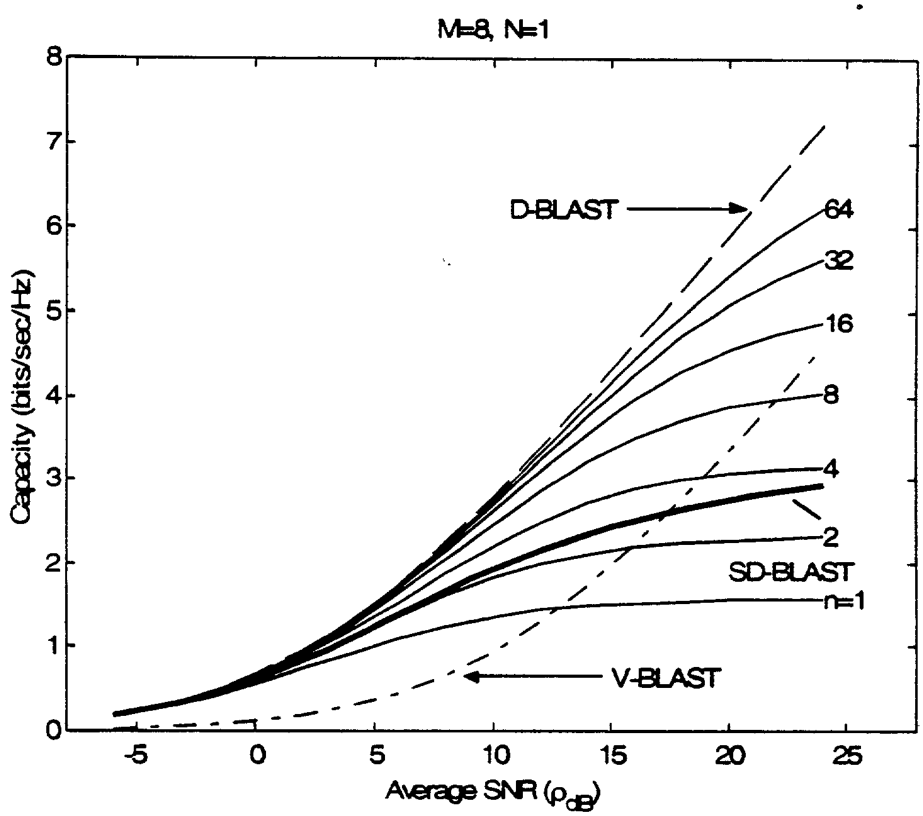
SD-BLAST EXAMPLE: (8,1)

The SD-BLAST graphs for 1, 2, 4, 8, 16, 32 and 64 strata per layer are shown in Figure 6A for the (8,1) example (as they are for the next two examples as well). For an (8,1) system the SD-BLAST graphs evidence D-BLAST, or what is the same, LogDet, performance as an upper envelope as predicted by the theory. Since the transmitter knows the channel statistics it knows the lower 10%-tile of $|\lambda_1|^2$. So, in this case, the single eigenrate at the

10% outage level is known to the transmitter. The bit rates per ply in the limit of large n are thereby known. Moreover, the guessing procedure was so effective here that the violation counts were negligible for all the n shown. So in this case the ply bit rates were essentially known. (This will not be the case in the subsequent examples.)

The lower double dash curve that appears in each figure expresses the performance of the upper limit of encoded V-BLAST systems. As expected, V-BLAST exhibits inferior performance.

Figure 6A: (8,1) SD-BLAST, CAPACITY vs AVERAGE SNR



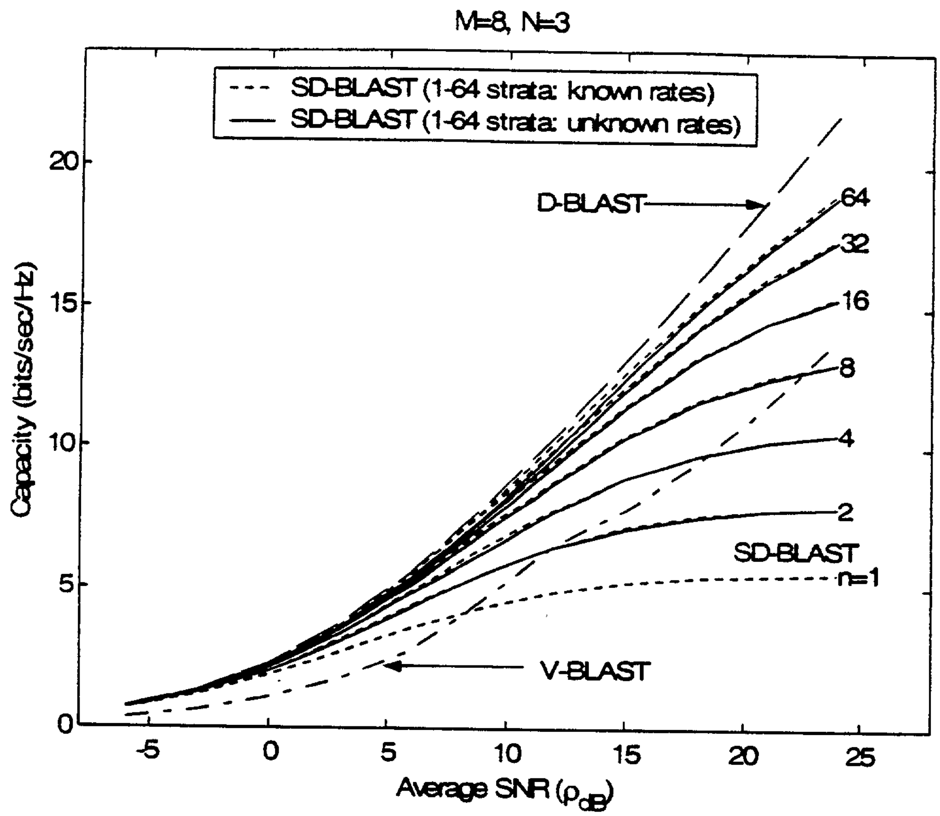
In this example, as well as in the three that follow, the n signal plies were equally energized. We mention in passing that one can get improved capacity at any given outage %-tile by optimizing the energy distribution over the n plies. To illustrate this, in this case, for n = 2 we optimized the energy distributed over the two plies and found considerable capacity improvement as Figure 6A shows with the bold curve. Of course, one can not improve over the upper envelope of LogDet performance, however, with (M,1) systems one can look to improve convergence to LogDet with increasing n by optimizing the distribution of the n signal energies.

SD-BLAST EXAMPLE: (8,3)

In this (8,3) case with M > N > 1, the problem is more difficult then the previous one since the transmitter does not know the set of n bit rates per strata to use in the limit of large n. The transmitter's guessing of the ply rates is now crucial. We cannot expect to get the same 10%-tiles as if the ply rates were known. The aforementioned guessing technique was used to produce the 10%-tile capacities. In Figure 6B, the degradation of these 10%-tiles from the 10%-tiles for when the n ply rates are known is shown and the degradations are just about noticeable.

The V-BLAST curves exhibit a peaking of the derivative. This is because (M,N) V-BLAST, requires, that, for each SNR reported, the optimum number of transmitters less than or equal to M was used and this optimum can change with the abscissa value. Such changes in the number of transmitters also occur in the other three examples, but in Figure 6A there was no noticeable peaking of the slope.

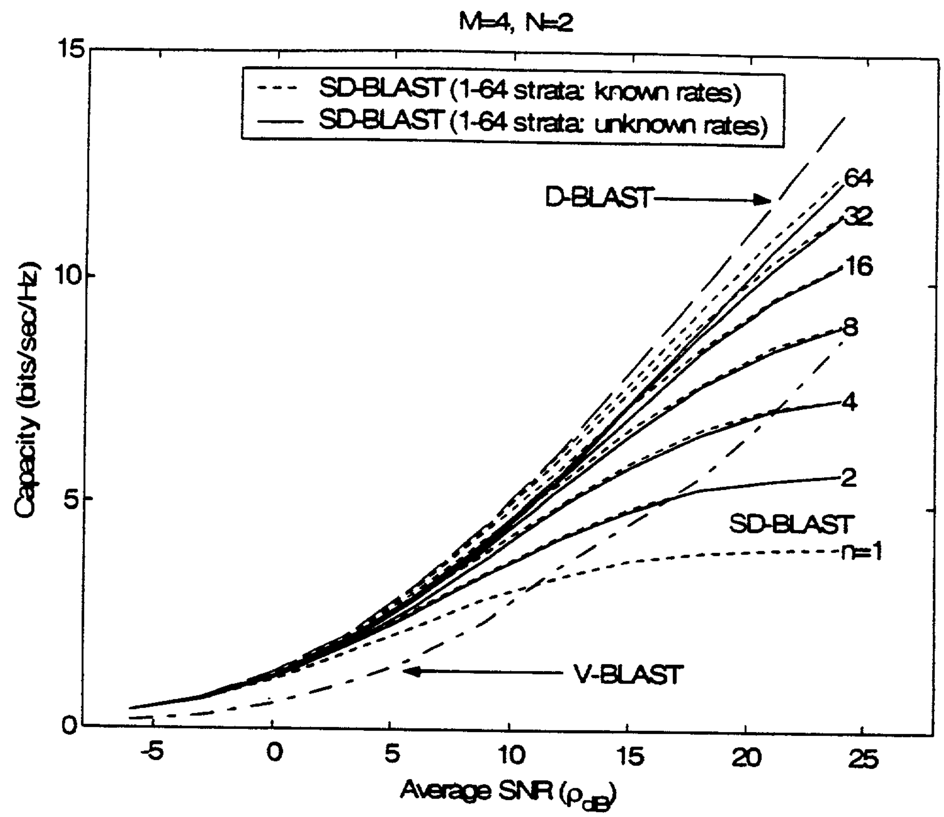
Figure 6B: (8,3) SD-BLAST, CAPACITY vs AVERAGE SNR



SD-BLAST EXAMPLE: (4,2)

In the third example, a (4,2) case depicted in Figure 6C, the excess of M over N is not as great as in the previous two examples. Therefore, as expected, the relative performance is not quite as good as the previous two cases. However, because of substantial eigenrate hardening, the deficit from known bit rates is still significantly less than one bps/Hz.

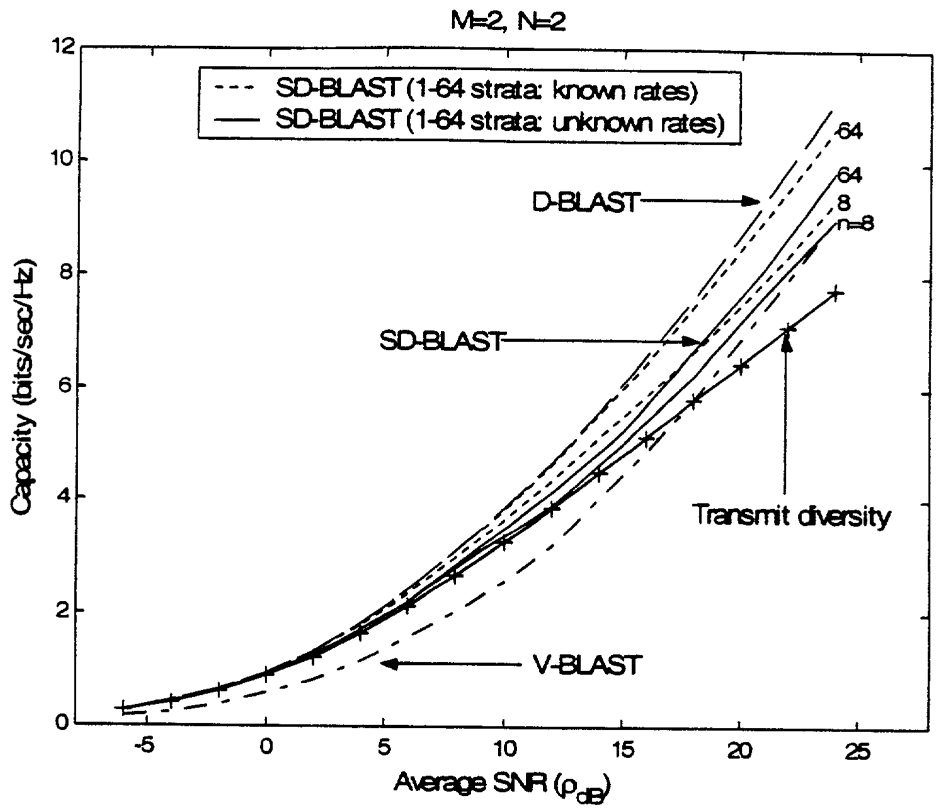
Figure 6C: (4,2) SD-BLAST, CAPACITY vs AVERAGE SNR



SD-BLAST EXAMPLE: (2,2)

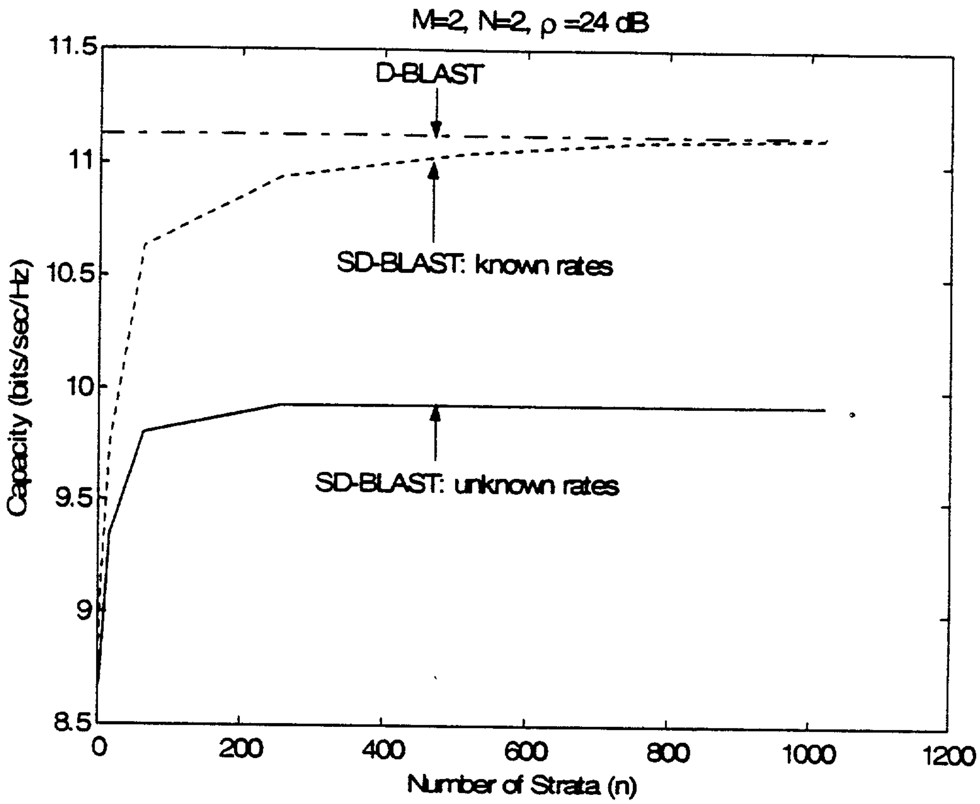
The fourth example, a (2,2) case in shown in Figure 6D. It is the most challenging of the four cases. To avoid overcrowding, only the eight and sixty four strata SD-BLAST curves are shown. There is a significant departure between what is possible with the ply bit rates known versus unknown. A transmit diversity scheme from reference [15], where the architectural superstructure requires two instances of 1-D encoding each with 2-D decoding is also shown. Even with eight strata SD-BLAST is seen to be competitive.

Figure 6D1: (2,2) SD-BLAST, CAPACITY vs AVERAGE SNR



The curves we have shown may make one wonder if the upper envelope of our genie unassisted examples is LogDet. The theory tells us that this cannot be for the (8,3), (4,2) and (2,2) examples. We looked for numerical confirmation of a deficit for this (2,2) example, at 24dB. We obtained results for a very large number of strata and indeed we noticed saturation below LogDet as the last Figure 6D2 shows.

Figure 6D2: (2,2) SD-BLAST, CAPACITY AT 24dB vs No. OF STRATA. EXPECTED SATURATION BELOW D-BLAST ASYMPTOTE



6 BRIEF COMMENTS ON VARIATIONS INCLUDING CDMA

There are straightforward variations on the covering of plies with helices. E.g., by perturbing the long message length, T, slightly so that T and M are relatively prime, one can arrange to uniformly wrap a ply with just one (approximately M times longer) helix rather than M parallel helices. This is meaningful when the code used has a time sense. In the course of detection processing along the single longer helix, when repeating the time of symbol receipt coordinate, merges must have been forced on the

simultaneously received, but previously processed helical section(s). In fact the entire message can be just one long helix by dropping down to the next inner ply to continue the ply wrapping after an outer ply is completely wrapped. In this case the received signal manifold can be taken to be like the snake who swallows his own tail along the lines depicted in [33].

As the number of plies increase, the variation of the previous two paragraphs are easily shown to be of secondary importance in terms of the capacity improvement afforded relative to the M-fold simultaneous detection featured for simplicity in Section 4.

One can replace the spatially 1-D codec constraint with, say, a spatially k-D codec constraint ($k \leq M$ and k dividing M). The 1-D choice in this report was the extreme low dimensional choice used for illustrative purposes.

The SD-BLAST approach also applies to frequency selective systems. For example, one can look to use a space-time message superstructure in accordance with OFDM features when communicating over a broadband frequency selective channel. A possible architecture is a closed loop chain of plied tori, one plied torus per subband and one or more per decorrelation band. Coding is over subbands. The chain link order would express the order in which the receiver's processing clock is associated with passage from one subband to the next. The standard frequency axis ordering is not to be respected since it is essential to pass very quickly through all decorrelated subbands and all antennas so that the stronger signal subbands can compensate weaker signal subbands at maximum rate. Ideally, the dwell on single strata would be only one symbol duration, then on to the corresponding stratum in the next torus in the chain. Thereby, bits

experience maximum space/frequency diversity and bit decisions are not held unnecessarily in abeyance.

With the exception of the last paragraph, the focus of this paper has been on any single one of many spectrally disjoint narrowband users. The numerical results presented in the last section go over immediately for any one of the orthogonal users in an idealized CDMA system based on orthogonal codes and operating over a flat channel. It is also possible to numerically assess performance of SD-BLAST CDMA systems using PN codes. Moreover, the SD-BLAST approach also applies to frequency selective CDMA systems.

7 SD-BLAST: SUMMARY AND FURTHER WORK

SD-BLAST is an architectural superstructure for an (M,N) communication system. Signal constituents are received symmetrically arranged in space-time in nested toroidal plies that are helically wound. They are so arranged to enable the receiver to substantially mute inevitable mutual interference of the M simultaneous transmissions in a multipath environment. At the same time SD-BLAST serves to enable implementation with 1-D codecs. It helps avoids practical problems associated with an earlier D-BLAST. Namely, it helps to avoid using long diagonals which force waste of the space-time resource and make it problematic to pack enough coded blocks into a message of limited duration.

The scope of this report does not include the lower level issue of 1-D codec design for the coded block occupying a stratum. With SD-BLAST, within a stratum, an SINR of period M is experienced. Reference [34,35] discusses coding for systems with periodically varying SNRs as arise in

ADSL systems. The authors stress that the signalling alphabet must be sufficiently rich to take advantage of the better SNRs. We note that, with SD-BLAST, with more and more plies, even a code using a binary constellation meets this important requirement because of the way that the bps/ply decrease with increasing n. Furthermore, the period M cycling at the transmitter has stronger transmissions compensating weaker ones at a maximum pace. This concentration of space diversity with the passage of processing time promotes quick delivery of bit decisions.

An information theory analysis showed that the SD-BLAST communication architecture can often get close to LogDet performance. Particularly we highlighted important downlink cases, mostly with M significantly greater than N, i.e., where the receiver has few antennas compared to the base(s). In illustrative (8,1), (8,3), (4,2) and (2,2) examples involving matrix Rayleigh channels we quantified the extent to which one can expect to get close to LogDet performance. Under the assumption of equal energy per stratum, Figures 6A-D quantified how much stratification was needed to support a particular capacity level at 10% outage for SNRs spanning –6dB to +24dB.

The simulation results reported in the literature, [36-39] (TURBO-BLAST architecture), [22] (Turbo Space-Time Architecture) and [40], (Threaded Space-Time Architecture) involving BLAST in conjunction with iterative receivers helped provide motivation for the work reported in our paper. These architectures do not include the element of stratification that we have explored that enables us to show that in some cases we can "attain" or come close to "attaining" LogDet performance in the limit of infinitely many strata.

Moreover, the SD-BLAST approach, theoretically analyzed here in the limit of zero error rate, was not iterative while the approaches in references of the previous paragraph were iterative. Of course, in practice, the SD-BLAST structure would operate under a specific bit or block error rate requirement. Using simulation, it would be interesting to quantify any advantages to including some iterative aspects of interstrata processing in SD-BLAST. Thus, one could see if it is worthwhile not to limit the processing to one pass through the various strata. Specific codes, like LDPC [41] and TURBO codes, can be tested in such SD-BLAST simulations. Ultimately, the most promising codes could be tested over the air using the Crawford Hill Wireless Research Department prototype.

It is worthwhile mentioning that we can conclude from [22-24] that LogDet can also be "attained" with separately coded 1-D signals radiating from each transmit antenna using codes that are designed for a standard AWGN environment. Each transmitter transmits a (possibly, depending on the channel) different rate. However, this requires that the M transmit rates be fed back to the transmitter.

We stressed the random channel case. Suppose instead the channel is not random, just that it is unknown to the transmitter. In this case the transmitter, beside N, and of course M, is only given the capacity of the fixed but otherwise completely unknown channel it is to communicate over. The question is: What capacity can be achieved with SD-BLAST when the transmitter is so blind? For fixed energy per ply and a large number of plies this is a very tractable optimization problem. The associated issue of optimizing the energy levels in a given number of plies is an interesting one to pursue for both the random and nonrandom channel cases.

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